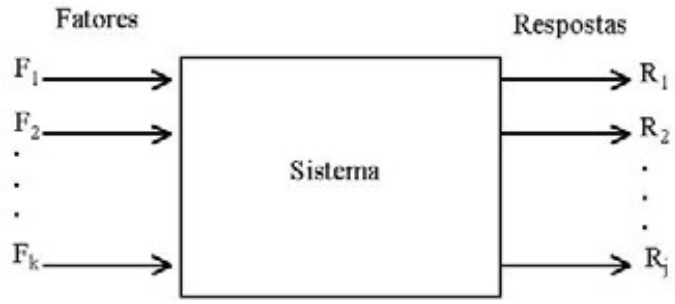
Como fazer experimentos



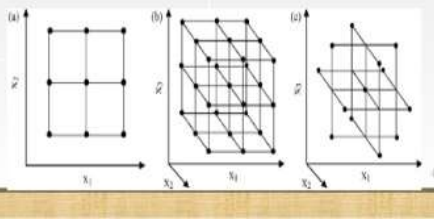
Representação de um sistema.

1

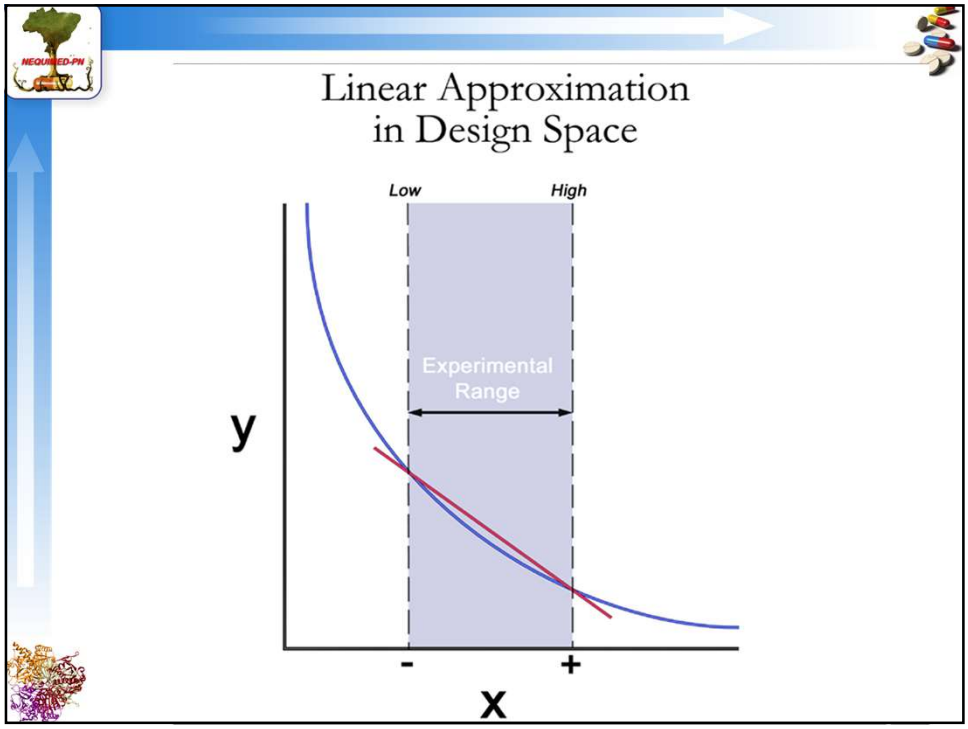
Types of Factorial Design

Factorial Design can be either

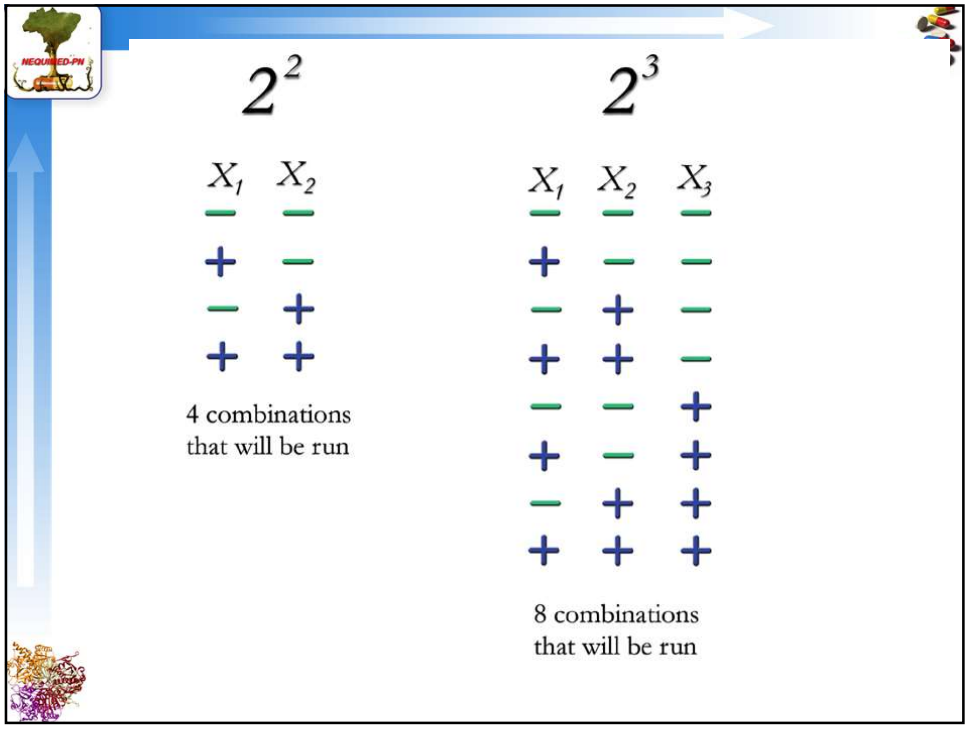
- Full FD
- Fractional FD



2


Linear Approximation
in Design Space
Low
High
Experimental
Range
y
-
+
x

3


2^2
X_1 X_2
4 combinations
that will be run
2^3
X_1 X_2 X_3
8 combinations
that will be run

4

Main Effect of X_i

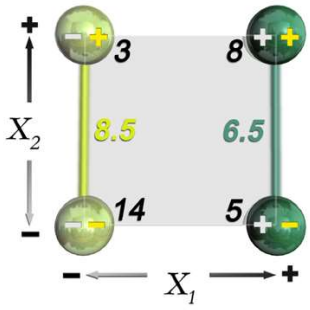
$(\bar{y}$ when X_i is high$) - (\bar{y}$ when X_i is low$)$

or in words,

$\left(\begin{array}{c}\text{average response} \\ \text{when } X_i \text{ is +}\end{array}\right) - \left(\begin{array}{c}\text{average response} \\ \text{when } X_i \text{ is -}\end{array}\right)$

5

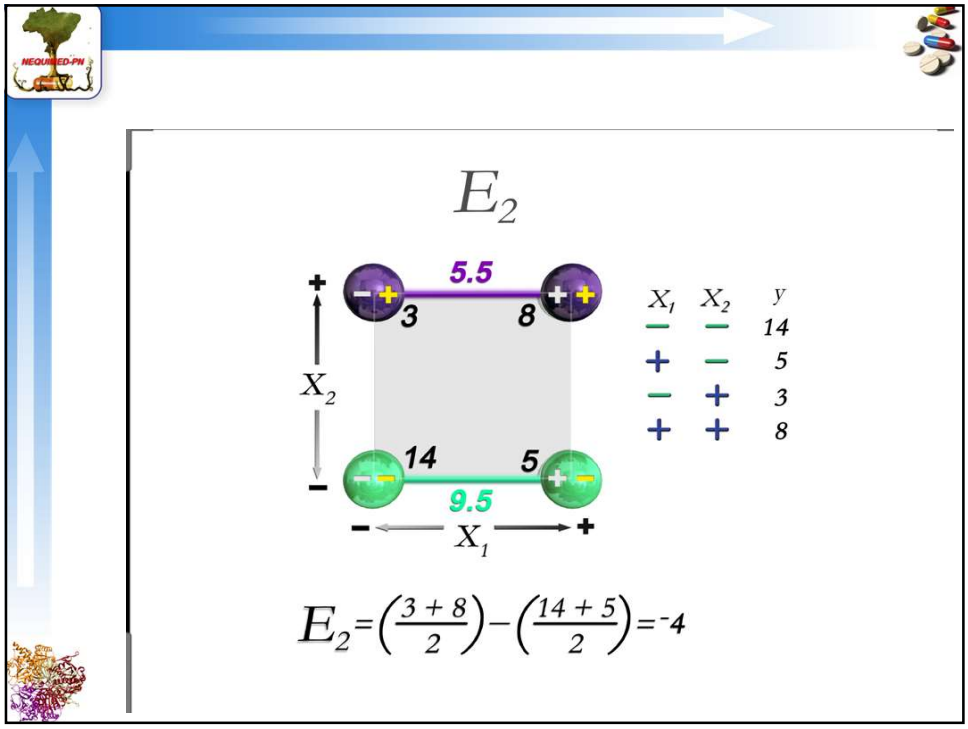
E_1



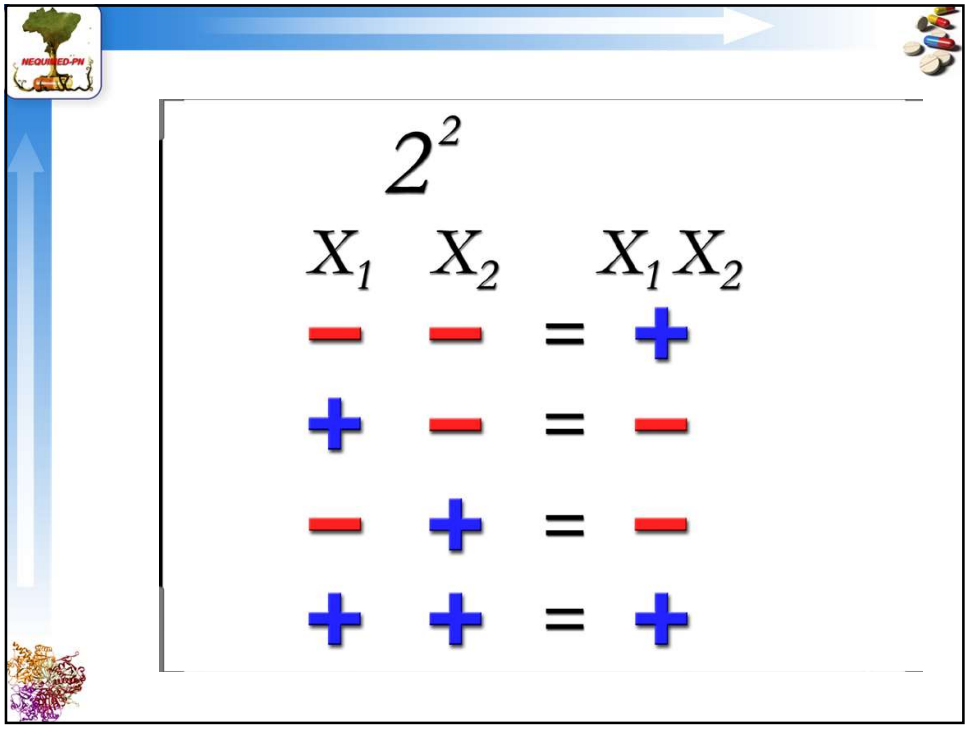
X_1	X_2	y
−	−	14
+	−	5
−	+	3
+	+	8

$$E_1 = \left(\frac{8+5}{2}\right) - \left(\frac{3+14}{2}\right) = -2$$

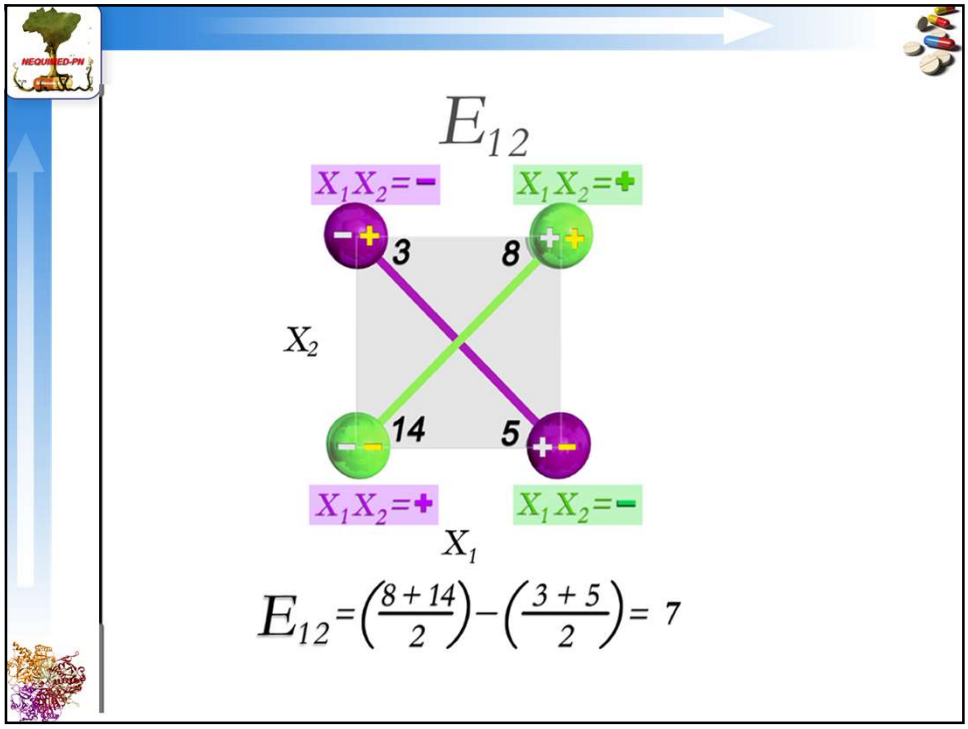
6


E_2
5.5
3
8
14
5
9.5
X_2
X_1
+
−
X_1 X_2 y
− − 14
+ − 5
− + 3
+ + 8
$E_2=\left(\frac{3+8}{2}\right)-\left(\frac{14+5}{2}\right)=-4$

7


2^2
X_1 X_2 X_1X_2
− − = +
+ − = −
− + = −
+ + = +

8



9

Design and Calculation Matrix

2^3 Experiment

X_1	X_2	X_3	X_1X_2	X_1X_3	X_2X_3	$X_1X_2X_3$
−	−	−	+	+	+	−
+	−	−	−	−	+	+
−	+	−	−	+	−	+
+	+	−	+	−	−	−
−	−	+	+	−	−	+
+	−	+	−	+	−	−
−	+	+	−	−	+	−
+	+	+	+	+	+	+

Design Matrix

Calculation Matrix

10

Calculation of E_1

X_1	X_2	X_3	X_1X_2	X_1X_3	X_2X_3	$X_1X_2X_3$	y
−	−	−	+	+	+	−	10
+	−	−	−	−	+	+	25
−	+	−	−	+	−	+	20
+	+	−	+	−	−	−	5
−	−	+	+	−	−	+	12
+	−	+	−	+	−	−	26
−	+	+	−	−	+	−	15
+	+	+	+	+	+	+	4

Average y when X_1 is +:

$$\frac{25 + 5 + 26 + 4}{4} = 15$$

Average y when X_1 is −:

$$\frac{10 + 20 + 12 + 15}{4} = 14.25$$

$$E_1 = 15 - 14.25 = 0.75$$

11

Planejamento fatorial 2^2
(Planejamento fatorial completo)

Ensaio	T(°C)	Catalisador	Rendimento(%)		Média
1	40	A	57	61	59
2	60	A	92	88	90
3	40	B	55	53	54
4	60	B	66	70	68

12

Normal

Standard Run	Design: 2**(2-0) design (bruns_2_2_FC.sta) F1 (Categ.)	F2 (Categ.)	DV_ 1
1	-1.00000	-1.00000	
2	1.00000	-1.00000	
3	-1.00000	1.00000	
4	1.00000	1.00000	

Aleatório

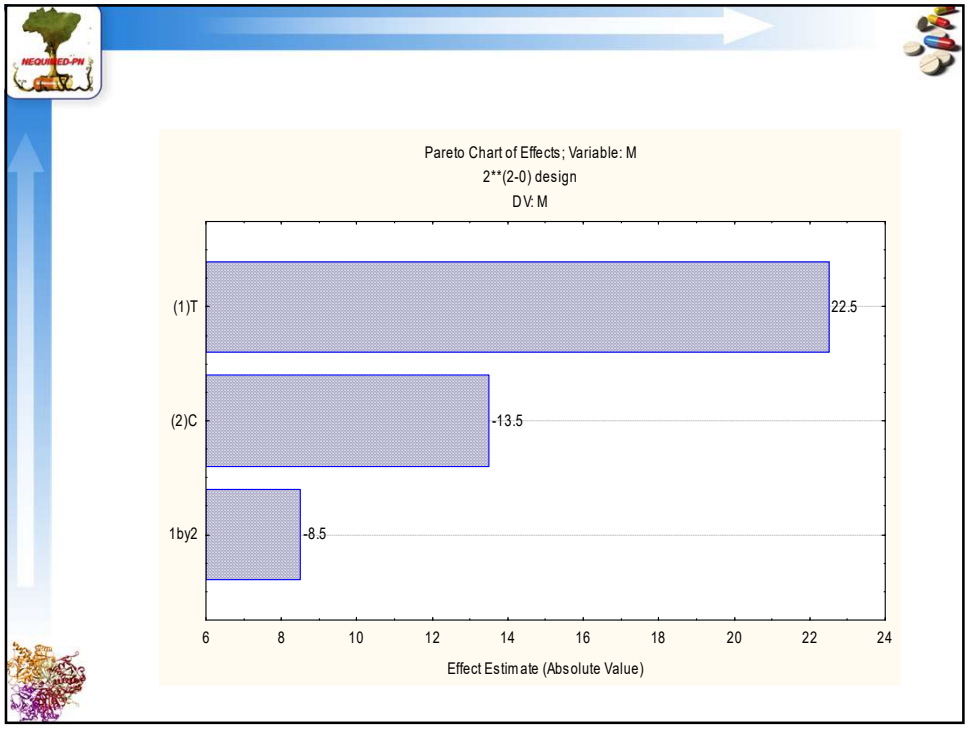
Standard Run	Design: 2**(2-0) design (bruns_2_2_FC.sta) F1 (Categ.)	F2 (Categ.)	DV_ 1
3	-1.00000	1.00000	
1	-1.00000	-1.00000	
4	1.00000	1.00000	
2	1.00000	-1.00000	

13

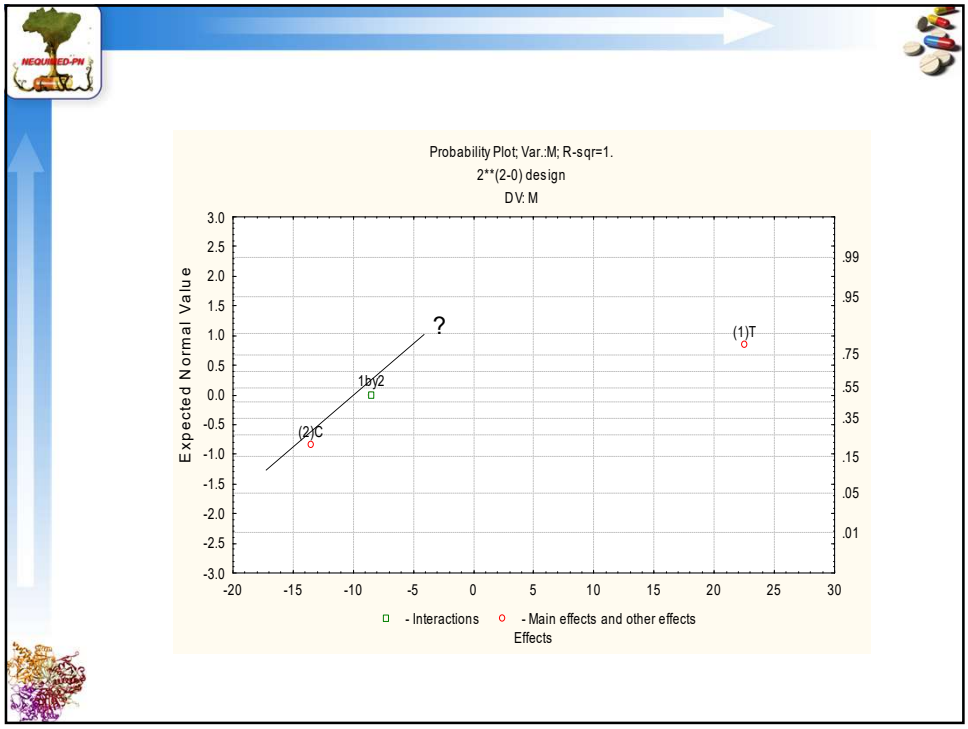
Cálculo dos Efeitos

Factor	Effect Estimates; Var.:M; R-sqr=1 2**(2-0) design DV: M Effect	Coeff.
Mean/Interc.	67.7500	67.75000
(1)T	22.5000	11.25000
(2)C	-13.5000	-6.75000
1 by 2	-8.5000	-4.25000

14


Pareto Chart of Effects; Variable: M
2**(2-0) design
DV: M
(1)T
(2)C
1by2
22.5
-13.5
-8.5
6
8
10
12
14
16
18
20
22
24
Effect Estimate (Absolute Value)

15


Probability Plot; Var.:M; R-sqr=1.
2**(2-0) design
DV: M
Expected Normal Value
3.0
2.5
2.0
1.5
1.0
0.5
0.0
-0.5
-1.0
-1.5
-2.0
-2.5
-3.0
.99
.95
.75
.55
.35
.15
.05
.01
?
(1)T
1by2
(2)C
-20
-15
-10
-5
0
5
10
15
20
25
30
- Interactions
- Main effects and other effects
Effects

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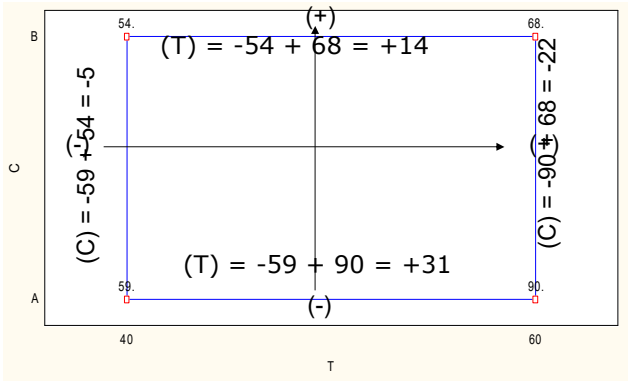
Interpretação geométrica dos Efeitos



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Interpretação dos resultados

Média global	67,8±0,9
Efeitos principais	
T	22,5±1,8
C	-13,5±1,8
Efeito de interação	
TC	-8,5±1,8

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Planejamento fatorial 2^4

Planejamento fatorial completo (Resolução completa)

	1 Var1	2 Var2	3 Var3	4 Var4	5 M
1	-1	-1	-1	-1	54
2	1	-1	-1	-1	85
3	-1	1	-1	-1	49
4	1	1	-1	-1	62
5	-1	-1	1	-1	64
6	1	-1	1	-1	94
7	-1	1	1	-1	56
8	1	1	1	-1	70
9	-1	-1	-1	1	52
10	1	-1	-1	1	87
11	-1	1	-1	1	49
12	1	1	-1	1	64
13	-1	-1	1	1	64
14	1	-1	1	1	94
15	-1	1	1	1	58
16	1	1	1	1	73

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Standard Run	Design: 2**(4-0) design (bruns_2_4_FC.sta)			
	A	B	C	D
4	1.00000	1.00000	-1.00000	-1.00000
15	-1.00000	1.00000	1.00000	1.00000
8	1.00000	1.00000	1.00000	-1.00000
10	1.00000	-1.00000	-1.00000	1.00000
16	1.00000	1.00000	1.00000	1.00000
1	-1.00000	-1.00000	-1.00000	-1.00000
3	-1.00000	1.00000	-1.00000	-1.00000
6	1.00000	-1.00000	1.00000	-1.00000
2	1.00000	-1.00000	-1.00000	-1.00000
9	-1.00000	-1.00000	-1.00000	1.00000
14	1.00000	-1.00000	1.00000	1.00000
11	-1.00000	1.00000	-1.00000	1.00000
7	-1.00000	1.00000	1.00000	-1.00000
5	-1.00000	-1.00000	1.00000	-1.00000
12	1.00000	1.00000	-1.00000	1.00000
13	-1.00000	-1.00000	1.00000	1.00000

20

Cálculo dos efeitos

Effect Estimates; Var.:M; R-sqr=.99984; Adj:.9976 (bruns_2_4_FC.sta)
2**(4-0) design; MS Residual=.5625
DV: M

Factor	Effect	Std.Err.	t(1)	p	-95.% Cnf.Limt	+95.% Cnf.Limt	Coeff.	Std.Err. Coeff.	-95.% Cnf.Limt	+95.% Cnf.Limt
Mean/Interc.	67.1875	0.187500	358.3333	0.001777	64.8051	69.56991	67.18750	0.187500	64.80509	69.56991
(1)Var1	22.8750	0.375000	61.0000	0.010435	18.1102	27.63983	11.43750	0.187500	9.05509	13.81991
(2)Var2	-14.1250	0.375000	-37.6667	0.016897	-18.8898	-9.36017	-7.06250	0.187500	-9.44491	-4.68009
(3)Var3	8.8750	0.375000	23.6667	0.026883	4.1102	13.63983	4.43750	0.187500	2.05509	6.81991
(4)Var4	0.8750	0.375000	2.3333	0.257762	-3.8898	5.63983	0.43750	0.187500	-1.94491	2.81991
1 by 2	-8.6250	0.375000	-23.0000	0.027662	-13.3898	-3.86017	-4.31250	0.187500	-6.69491	-1.93009
1 by 3	-0.6250	0.375000	-1.6667	0.344042	-5.3898	4.13983	-0.31250	0.187500	-2.69491	2.06991
1 by 4	0.8750	0.375000	2.3333	0.257762	-3.8898	5.63983	0.43750	0.187500	-1.94491	2.81991
2 by 3	-0.6250	0.375000	-1.6667	0.344042	-5.3898	4.13983	-0.31250	0.187500	-2.69491	2.06991
2 by 4	0.8750	0.375000	2.3333	0.257762	-3.8898	5.63983	0.43750	0.187500	-1.94491	2.81991
3 by 4	0.3750	0.375000	1.0000	0.500000	-4.3898	5.13983	0.18750	0.187500	-2.19491	2.56991
1*2*3	0.8750	0.375000	2.3333	0.257762	-3.8898	5.63983	0.43750	0.187500	-1.94491	2.81991
1*2*4	-0.1250	0.375000	-0.3333	0.795167	-4.8898	4.63983	-0.06250	0.187500	-2.44491	2.31991
1*3*4	-0.6250	0.375000	-1.6667	0.344042	-5.3898	4.13983	-0.31250	0.187500	-2.69491	2.06991
2*3*4	0.3750	0.375000	1.0000	0.500000	-4.3898	5.13983	0.18750	0.187500	-2.19491	2.56991

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Coeficientes de regressão

Regr. Coefficients; Var.:M; R-sqr=.99984; Adj:.9976 (bruns_2_
2**(4-0) design; MS Residual=.5625
DV: M

Factor	Regressn Coeff.	Std.Err.	t(1)	p	-95.% Cnf.Limt	+95.% Cnf.Limt
Mean/Interc.	67.18750	0.187500	358.3333	0.001777	64.80509	69.56991
(1)Var1	11.43750	0.187500	61.0000	0.010435	9.05509	13.81991
(2)Var2	-7.06250	0.187500	-37.6667	0.016897	-9.44491	-4.68009
(3)Var3	4.43750	0.187500	23.6667	0.026883	2.05509	6.81991
(4)Var4	0.43750	0.187500	2.3333	0.257762	-1.94491	2.81991
1 by 2	-4.31250	0.187500	-23.0000	0.027662	-6.69491	-1.93009
1 by 3	-0.31250	0.187500	-1.6667	0.344042	-2.69491	2.06991
1 by 4	0.43750	0.187500	2.3333	0.257762	-1.94491	2.81991
2 by 3	-0.31250	0.187500	-1.6667	0.344042	-2.69491	2.06991
2 by 4	0.43750	0.187500	2.3333	0.257762	-1.94491	2.81991
3 by 4	0.18750	0.187500	1.0000	0.500000	-2.19491	2.56991
1*2*3	0.43750	0.187500	2.3333	0.257762	-1.94491	2.81991
1*2*4	-0.06250	0.187500	-0.3333	0.795167	-2.44491	2.31991
1*3*4	-0.31250	0.187500	-1.6667	0.344042	-2.69491	2.06991
2*3*4	0.18750	0.187500	1.0000	0.500000	-2.19491	2.56991

pH

$Y = 11,43(T) – 7,06(C) + 4,44(Conc) + 67,19$

22

Multiple Linear Regression Equations

A4

23

Planejamento fatorial fracionário (2^7 = 128 ensaios!)

Efeitos de interações não significativos?

Efeitos principais não significativos?

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Full Factorials

Number Factors	Main Effects	Order of Interactions								
		2	3	4	5	6	7	8	9	10
2	2	1								
3	3	3	1							
4	4	6	4	1						
5	5	10	10	5	1					
6	6	15	20	15	6	1				
7	7	21	35	35	21	7	1			
8	8	28	56	70	56	28	8	1		
9	9	36	84	126	126	84	36	9	1	
10	10	45	120	210	252	210	120	45	10	1

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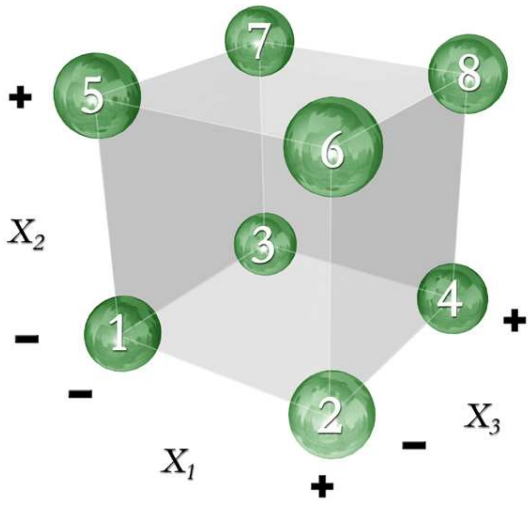
Selecting 4 of the 8



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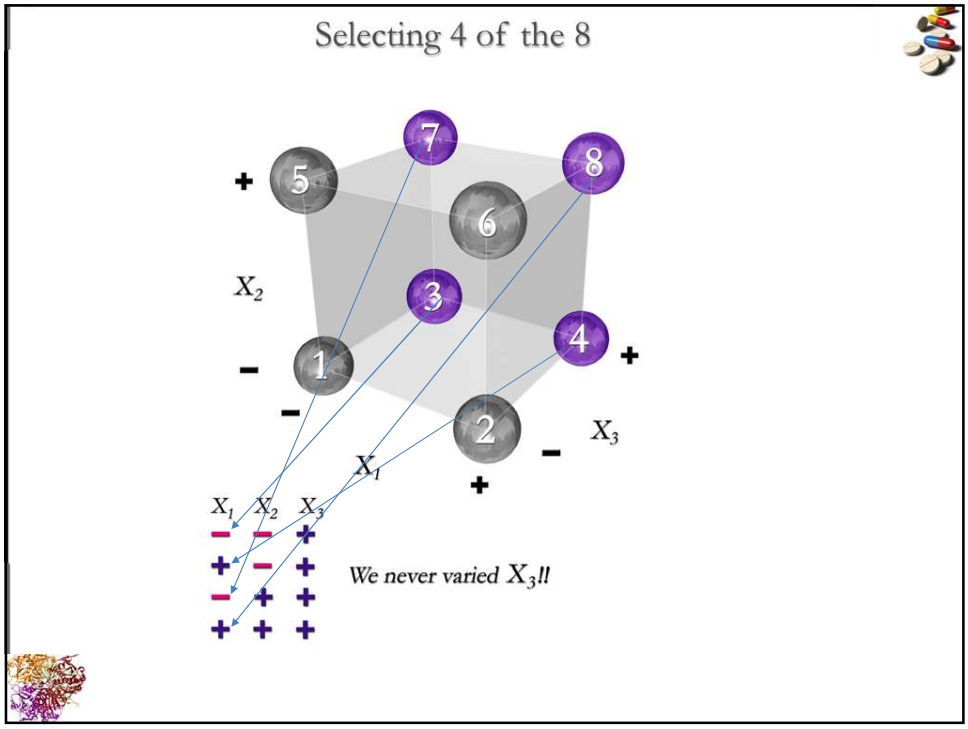
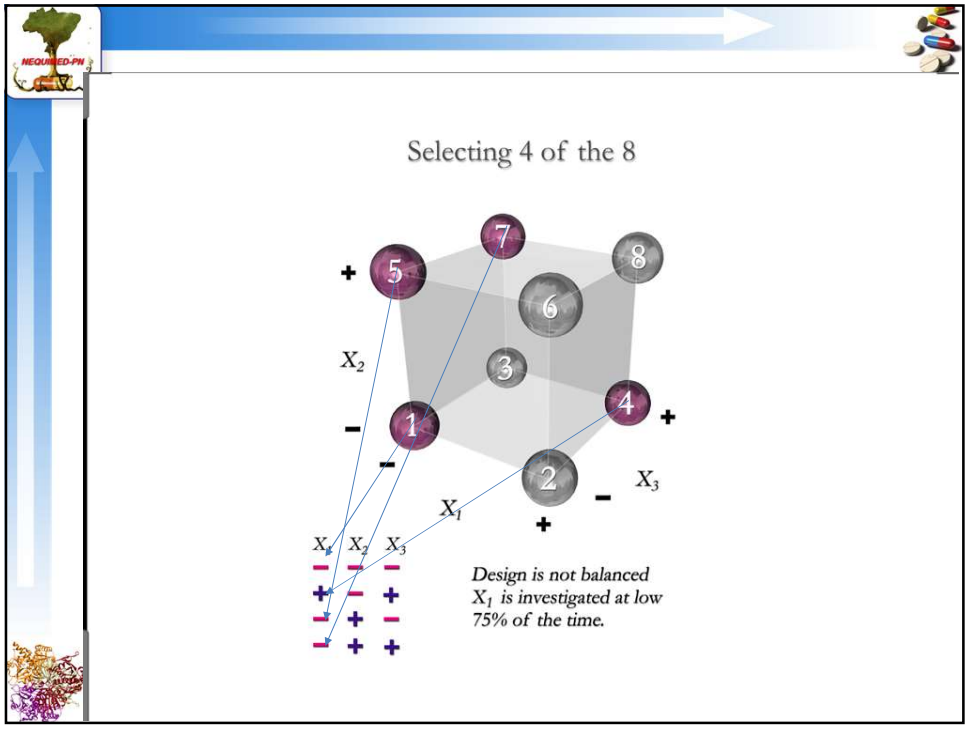

Selecting 4 of the 8
7
8
5
6
X_2
3
4
1
2
X_3
X_1
X_1 X_2 X_3
We never varied X_3!!

27


Selecting 4 of the 8
7
8
5
6
X_2
3
4
1
2
X_3
X_1
X_1 X_2 X_3
Design is not balanced
X_1 is investigated at low
75% of the time.

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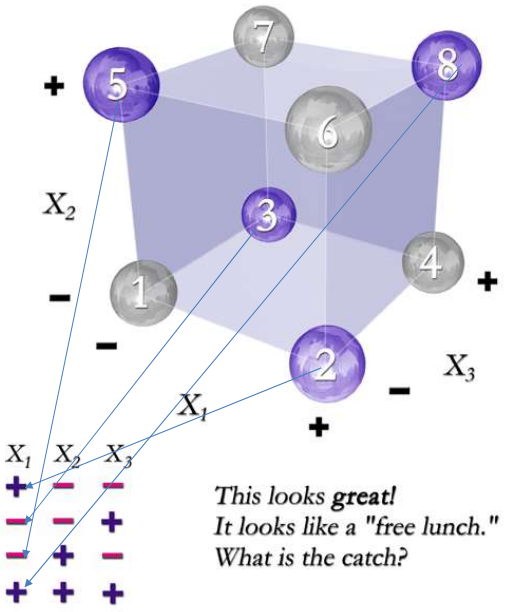
Regular Fractional Factorial Designs



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Fractional Factorial Notation

$$2_R^{k-p}$$

- "2" indicates each factor has two levels
- "k" indicates the number of factors included
- "p" indicates the fraction to be run
  "p" also indicates the number of "extra" factors that need to be placed into the base design
- "R" indicates the resolution of the design

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Planejamento fatorial 2^4
Como realizar apenas 8 ensaios?

	1 Var1	2 Var2	3 Var3	4 Var4	5 M
1	-1	-1	-1	-1	52
2	1	-1	-1	-1	61
3	-1	1	-1	-1	124
4	1	1	-1	-1	113
5	-1	-1	1	-1	85
6	1	-1	1	-1	60
7	-1	1	1	-1	185
8	1	1	1	-1	192
9	-1	-1	-1	1	98
10	1	-1	-1	1	86
11	-1	1	-1	1	201
12	1	1	-1	1	194
13	-1	-1	1	1	122
14	1	-1	1	1	139
15	-1	1	1	1	289
16	1	1	1	1	286

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Standard Run	Design: 2**(4-1) design (bruns_2_4_FF_ A	B	C	D
2	1.00000	-1.00000	-1.00000	1.00000
1	-1.00000	-1.00000	-1.00000	-1.00000
6	1.00000	-1.00000	1.00000	-1.00000
5	-1.00000	-1.00000	1.00000	1.00000
8	1.00000	1.00000	1.00000	1.00000
7	-1.00000	1.00000	1.00000	-1.00000
4	1.00000	1.00000	-1.00000	-1.00000
3	-1.00000	1.00000	-1.00000	1.00000

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Como construir um planejamento FF 2^{4-1}

1. Construir um planejamento 2^3 completo para os fatores 1, 2, e 3

2. Atribuir ao fator 4, os sinais dos produtos de 1, 2, e 3

4 = 123

	1	2	3	4	5 Y
1	-1	-1	-1	-1	52
2	1	-1	-1	1	86
3	-1	1	-1	1	201
4	1	1	-1	-1	113
5	-1	-1	1	1	122
6	1	-1	1	-1	66
7	-1	1	1	-1	185
8	1	1	1	1	286

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Comparação dos valores dos efeitos

2^{4-1}

Factor	Effect	Std.Err.	t(3)	p	-95.% Cnf.Limt	+95.% Cnf.Limt
Mean/Interc.	138.8750	10.81930	12.83586	0.001020	104.4432	173.3068
(1)V1	-2.2500	21.63860	-0.10398	0.923746	-71.1137	66.6137
(2)V2	114.7500	21.63860	5.30302	0.013089	45.8863	183.6137
(3)V3	51.7500	21.63860	2.39156	0.096606	-17.1137	120.6137
(4)V4	69.7500	21.63860	3.22341	0.048458	0.8863	138.6137

2^4

Factor	Effect	Std.Err.	t(5)	p	-95.% Cnf.Limt	+95.% Cnf.Limt
Mean/Interc.	143.3125	2.464276	58.15603	0.000000	136.9779	149.6471
(1)Var1	-2.3750	4.928552	-0.48189	0.650240	-15.0442	10.2942
(2)Var2	109.3750	4.928552	22.19212	0.000003	96.7058	122.0442
(3)Var3	54.3750	4.928552	11.03265	0.000107	41.7058	67.0442
(4)Var4	67.1250	4.928552	13.61962	0.000038	54.4558	79.7942
1 by 2	-1.1250	4.928552	-0.22826	0.828484	-13.7942	11.5442
1 by 3	2.8750	4.928552	0.58334	0.584987	-9.7942	15.5442
1 by 4	1.1250	4.928552	0.22826	0.828484	-11.5442	13.7942
2 by 3	25.6250	4.928552	5.19930	0.003469	12.9558	38.2942
2 by 4	21.8750	4.928552	4.43842	0.006775	9.2058	34.5442
3 by 4	9.8750	4.928552	2.00363	0.101468	-2.7942	22.5442

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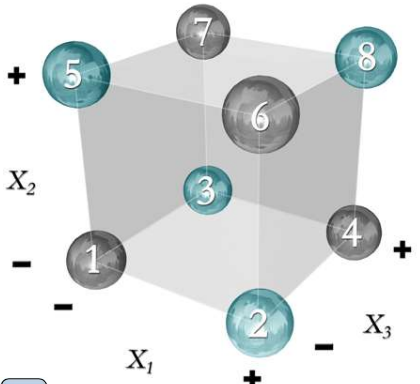
Confounding or Aliasing
→ NO FREE LUNCH!!!



X_1	X_2	X_3	X_1X_2	X_1X_3	X_2X_3
+	−	−	−	−	+
−	−	+	+	−	−
−	+	−	−	+	−
+	+	+	+	+	+

Who caused the effect? X_2 or the X_1X_3 interaction? Each main effect is ***CONFOUNDED*** *with a two factor interaction!!*

X3 = X1X2 → X1X3 = X2 and X2X3 = X1
(main effects aliased with two-factor interactions) – Resolution III design

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Design Generators and Resolution: 2^{6-2}

X5 = X1*X2*X3; X6 = X2*X3*X4 → X5*X6 = X1*X4

5 = 123; 6 = 234; 56 = 14 →

Generators: I = 1235 = 2346 = 1456

Resolution: Length of the shortest "word" in the generator set → resolution IV here

So …

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Resolution

Resolution III: (1+2)

Main effect aliased with 2-order interactions

Resolution IV: (1+3 or 2+2)

Main effect aliased with 3-order interactions and 2-factor interactions aliased with other 2-factor …

Resolution V: (1+4 or 2+3)

Main effect aliased with 4-order interactions and 2-factor interactions aliased with 3-factor interactions

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¼ fraction of $2^6 = 2^{6-2}$ FFD

Place x5 Here

Place x6 Here

x1	x2	x3	x4	x1x2	x1x3	x1x4	x2x3	x2x4	x3x4	x1x2x3	x1x2x4	x1x3x4	x2x3x4	x1x2x3x4
-	-	-	-	+	+	+	+	+	+	-	-	-	-	+
+	-	-	-	-	-	-	+	+	+	+	+	+	-	-
-	+	-	-	-	+	+	-	-	+	+	+	-	+	-
+	+	-	-	+	-	-	-	-	+	-	-	+	+	+
-	-	+	-	+	-	+	-	+	-	+	-	+	+	-
+	-	+	-	-	+	-	-	+	-	-	+	-	+	+
-	+	+	-	-	-	+	+	-	-	-	+	+	-	+
+	+	+	-	+	+	-	+	-	-	+	-	-	-	-
-	-	-	+	+	+	-	+	-	-	-	+	+	+	-
+	-	-	+	-	-	+	+	-	-	+	-	-	+	+
-	+	-	+	-	+	-	-	+	-	+	-	+	-	+
+	+	-	+	+	-	+	-	-	-	-	+	-	-	-
-	-	+	+	+	-	-	-	-	+	+	-	-	-	+
+	-	+	+	-	+	+	-	-	+	-	+	-	+	-
-	+	+	+	-	-	-	+	+	+	-	-	+	+	-
+	+	+	+	+	+	+	+	+	+	+	+	+	+	+

X5 = X2*X3*X4; X6 = X1*X2*X3*X4; → X5*X6 = X1

or I = 2345 = 12346 = 156 → Resolution III design

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2^{6-2} Experiment

x1	x2	x3	x4	x1x2	x1x3	x1x4	x2x3	x2x4	x3x4	x1x2x3 (Place x5 Here)	x1x2x4	x1x3x4	x2x3x4 (Place x6 Here)	x1x2x3x4
-	-	-	-	+	+	+	+	+	+	-	-	-	-	+
+	-	-	-	-	-	-	+	+	+	+	+	+	-	-
-	+	-	-	-	+	+	-	-	+	+	+	-	+	-
+	+	-	-	+	-	-	-	-	+	-	-	+	+	+
-	-	+	-	+	-	+	-	+	-	+	-	+	+	-
+	-	+	-	-	+	-	-	+	-	-	+	-	+	+
-	+	+	-	-	-	+	+	-	-	-	+	+	-	+
+	+	+	-	+	+	-	+	-	-	+	-	-	-	-
-	-	-	+	+	+	-	+	-	-	-	+	+	+	-
+	-	-	+	-	-	+	+	-	-	+	-	-	+	+
-	+	-	+	-	+	-	-	+	-	+	-	+	-	+
+	+	-	+	+	-	+	-	+	-	-	+	-	-	-
-	-	+	+	+	-	-	-	-	+	+	+	-	-	+
+	-	+	+	-	+	+	-	-	+	-	-	+	-	-
-	+	+	+	-	-	-	+	+	+	-	-	-	+	-
+	+	+	+	+	+	+	+	+	+	+	+	+	+	+

X5 = X1*X2*X3; X6 = X2*X3*X4 → X5*X6 = X1*X4

or I = 1235 = 2346 = 1456 → Resolution IV design

(after Vijay Nair)

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